

the feed back control $u = -Kx + lr$

closed loop system will have:

- Zero-steady-state error for a unit-step input
- 5% overshoot and 2 seconds settling time.

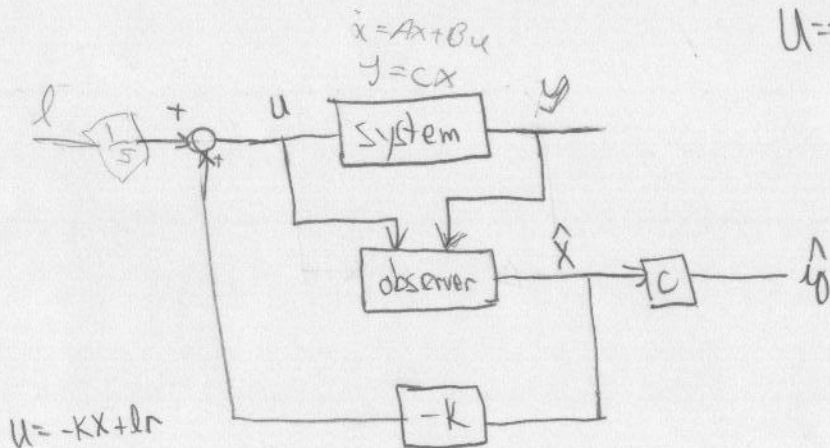
c) Give the block diagram of the observer-based state feedback controller.

Percent overshoot: $\%OS = 100 \exp\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)$

Settling time at 2%: $T_s \approx \frac{4}{\xi\omega_n}$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(\hat{y} - y)$$

$$u = -K\hat{x} + lr$$



$$u = -Kx + lr$$

$$\dot{\hat{x}} = A\hat{x} + Bu$$

$$y = Cx$$

$$\dot{\hat{x}} = A\hat{x} - BK\hat{x} + Blr$$

$$\dot{\hat{x}} = (A - BK)\hat{x} + Blr$$

$$sX(s) - X(s)[A - BK] = Blr(s)$$

$$X(s)[sI - (A - BK)] = Blr(s)$$

$$X(s) = [sI - (A - BK)]^{-1} Blr(s)$$

$$Y(s) = C[sI - (A - BK)]^{-1} Blr(s)$$

$$\lim_{y \rightarrow 0} = \lim_{s \rightarrow 0} Y(s) = C[sI - (A - BK)]^{-1} B \frac{1}{s} = 1$$

$$Q = [C(BK - A)^{-1}B]^{-1}$$

Eng. 0138: Advanced Control II

March 4, 2004.

8:30 AM to 10:00 AM

Closed Book exam.

Programmable calculators are not allowed.

Q-1 (8 marks)

Consider the following system

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \ 0]$$

a) Write the system in controllable canonical form, observable canonical form and diagonal canonical form or Jordan canonical form.

Q-2 (8 marks)

Consider the following matrices

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, A_2 = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Compute $e^{A_1 t}$ and $e^{A_2 t}$.

Q-3 (14 marks)

Consider the system defined by:

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

where,

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = [1 \ 0]$$

a) Check the state controllability and observability of the system.

$$U(s) \rightarrow \boxed{} \rightarrow X(s)$$

$$SX(s) = AX(s) + BU(s)$$

$$X(s)[sI - A] = BU(s)$$

$$\frac{X(s)}{U(s)} = [sI - A]^{-1} B U(s)$$

$$Y = CX$$

$$\frac{Y(s)}{U(s)} = C[sI - A]^{-1} B$$

$$U(s)$$

$$\det(sI - A) = 0$$

$$\lambda(\lambda - 1) = 0$$

$$2.97$$

$$0.6899$$

$$2 = \frac{4}{\omega_n(\zeta\omega_n)}$$

$$\omega_n = \frac{4}{2(0.6899)}$$

$$\omega_n = 2.97$$

$$\xi + \eta = 8.7012$$

$$Y(s) = C(sI - A)^{-1}B.$$

$$\rightarrow (sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} s-2 & -1 \\ -4 & s-3 \end{bmatrix}$$

$$\det(sI - A) = (s-2)(s-3) - 4 = s^2 - 5s + 6 - 4$$

$$\det(sI - A) = s^2 - 5s + 2$$

$$(sI - A)^{-1} \rightarrow \text{adj} \rightarrow \begin{matrix} C_{11} = s-3 & C_{12} = 4 \\ C_{21} = 1 & C_{22} = s-2 \end{matrix}$$

$$\therefore \text{adj}(sI - A) \rightarrow \begin{bmatrix} s-3 & 1 \\ 4 & s-2 \end{bmatrix}$$

$$\therefore Y(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s-3 & 1 \\ 4 & s-2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{s^2 - 5s + 2}$$

$$\frac{Y(s)}{U(s)} = \begin{bmatrix} s-3 & 1 \\ 4 & s-2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{Y(s)}{U(s)} = \frac{1}{s^2 - 5s + 2}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2} \rightarrow \frac{5 \pm \sqrt{25 - 8}}{2} = \frac{5 \pm \sqrt{17}}{2}$$

$$\therefore \text{poles are } \left(\frac{5}{2} + i\sqrt{17}/2 \right), \left(\frac{5}{2} - i\sqrt{17}/2 \right)$$

$$4.561, 0.4389$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2 - 5s + 2} = \frac{C_1}{(s - \frac{5}{2} + i\sqrt{17}/2)} + \frac{C_2}{(s - \frac{5}{2} - i\sqrt{17}/2)}$$

controllable

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(9)

OBSERVABLE

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

$$\begin{matrix} s = \frac{5}{2} + i\sqrt{17}/2 \\ s = \frac{5}{2} - i\sqrt{17}/2 \end{matrix}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{Let } 1 = C_1(s - \frac{5}{2} - i\sqrt{17}/2) + C_2(s - \frac{5}{2} + i\sqrt{17}/2)$$

DIAGONAL

$$\text{Let } s = \frac{5}{2} + i\sqrt{17}/2$$

$$\text{Let } s = \frac{5}{2} - i\sqrt{17}/2$$

$$1 = C_2(\frac{5}{2} + i\sqrt{17}/2 - \frac{5}{2} - i\sqrt{17}/2)$$

$$1 = C_1(\frac{5}{2} - i\sqrt{17}/2 - \frac{5}{2} + i\sqrt{17}/2)$$

$$1 = C_2 \sqrt{17}$$

$$1 = -\sqrt{17} C_1$$

$$Y/\sqrt{17} = C_2$$

$$-1/\sqrt{17} = C_1$$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} + i\sqrt{17}/2 & 0 \\ 0 & \frac{5}{2} - i\sqrt{17}/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \quad y = \begin{bmatrix} -1/\sqrt{17} & \sqrt{17} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Q-2) $A_1 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

$(\lambda I - A_1) = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} \lambda & -1 \\ 2 & \lambda+3 \end{bmatrix}$

Let $\lambda(\lambda+3)+2 = \lambda^2 + 3\lambda + 2 = (\lambda+1)(\lambda+2)$.

$\therefore \lambda = -1, -2$ Since Real, use P, P^{-1} method

$\therefore P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}, P^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

and $e^{A_1 t} = \begin{bmatrix} e^{-t} & \\ & e^{-2t} \end{bmatrix}$

$\therefore e^{A_1 t} = P e^{A_1 t} P^{-1} = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

$e^{A_1 t} = \begin{bmatrix} e^{-t} & -e^{-2t} \\ -e^{-t} & -2e^{-2t} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

$e^{A_1 t} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix} = \begin{bmatrix} e^{-t}(2 - e^{-t}) & e^{-t}(1 - e^{-t}) \\ 2e^{-t}(e^{-t} - 1) & e^{-t}(2e^{-t} - 1) \end{bmatrix}$

\Rightarrow THIS IS IN JORDAN FORM;

$A_2 = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$

$\therefore e^{A_2 t} = \begin{bmatrix} e^{3t} & te^{3t} & \frac{t^2}{2}e^{3t} & 0 & 0 \\ 0 & e^{3t} & te^{3t} & 0 & 0 \\ 0 & 0 & e^{3t} & 0 & 0 \\ 0 & 0 & 0 & e^{4t} & 0 \\ 0 & 0 & 0 & 0 & e^{5t} \end{bmatrix}$

(9)

(8-3)

$$a) \quad G = \begin{bmatrix} B & A & B \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 4 \end{bmatrix} \quad \det = -1 \quad \therefore \text{ITS NONSINGULAR}$$

$$\text{Rank}(G) = 2 \quad \therefore \text{ITS STATE CONTROLLABLE}$$

$$\text{Obs} \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \quad \det 1 - 0 = 1 \quad \therefore \text{ITS NONSINGULAR}$$

$$\text{Rank}(\text{Obs}) = 2 \quad \therefore \text{IT IS OBSERVABLE}$$

b) DESIGN K; \rightarrow

$$9.0\% = 100 \exp\left\{-\frac{\xi \omega_n}{\sqrt{1-\xi^2}}\right\}$$

$$T_s = \frac{4}{\omega_n \xi}$$

$$\therefore \xi = \frac{\ln^2(0.05)}{\sqrt{\ln^2(0.05) + \pi^2}} \quad \omega_n = \frac{4}{2(0.6901)} = \underline{2.898}$$

$$\xi = \underline{0.6901}$$

$$\therefore \text{DESIGN CHAR. GAIN} \quad \begin{aligned} & s^2 + 2\xi\omega_n s + \omega_n^2 \\ & s^2 + 2(0.6901)(2.898)s + (2.898)^2 \\ & \rightarrow s^2 + 4s + 8.398 \end{aligned}$$

$$\text{Finding } K; \quad y(s) = C[sI - (A - BK)]^{-1} B/r$$

$$\det(sI - A + BK)$$

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [k_1 \ k_2]$$

$$\therefore \begin{bmatrix} s-2 & -1 \\ -3 & s-4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}$$

$$\begin{bmatrix} s-2 & -1 \\ k_1-3 & s-4+k_2 \end{bmatrix}$$

$$\begin{aligned} \det(sI - A + BK) &= (s-2)(s-4+k_2) + (k_1-3) \\ &= s^2 - 4s + k_2s - 2s + 8 - 2k_2 + k_1 - 3 \\ &= s^2 + s(k_2 - 4 - 2) + 8 - 2k_2 + k_1 - 3 \end{aligned}$$

$$\therefore \text{Matching} \rightarrow s^2 + s(k_2 - 6) - 8 - 2k_2 + k_1 - 3 = s^2 + 4s + 8.39$$

$$\begin{aligned} \therefore k_2 - 6 &= 4 & 8 - 2(10) + k_1 - 3 &= 8.39 \\ k_2 &= 10 & -12 + k_1 - 3 &= 8.39 \\ & & k_1 &= 8.39 + 3 + 12 \\ & & k_1 &= 23.39 \end{aligned}$$

$$\therefore K = \begin{bmatrix} 23.39 & 10 \end{bmatrix}$$



finding l

$$y(s) = C [sI - A + BK]^{-1} B/r$$

Step

$$\lim_{t \rightarrow \infty} y(t) = 1 \Rightarrow \lim_{s \rightarrow 0} s y(s) = 1$$

$$\lim_{s \rightarrow 0} C [sI - A + BK]^{-1} B/r = C [-A + BK]^{-1} B/r = 1$$

$$C [BK - A]^{-1} B/r = 1$$

$$l = \{ C [BK - A]^{-1} B \}^{-1}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 23.39 & 10 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 20.39 & 6 \end{bmatrix} \quad \det. (12.11.11)$$

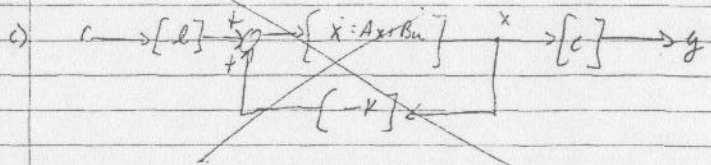
$$(BK - A)^{-1} = \begin{bmatrix} 0.715 & 0.119 \\ -2.43 & 0.238 \end{bmatrix}$$

$$l = \left\{ \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0.715 & 0.119 \\ -2.43 & 0.238 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}^{-1}$$

$$l = \left\{ \begin{bmatrix} 0.119 \end{bmatrix} \right\}^{-1} \quad \boxed{l = 8.40331}$$

✓

8.40336



2004 #1

Q1

given

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\dot{X} = AX + BU$$

$$SX(s) = AX(s) + BU(s)$$

$$SX(s) - AX(s) = BU(s)$$

$$X(s)[SI - A] = BU(s)$$

$$X(s) = [SI - A]^{-1} BU(s)$$

must find
T.F.

$$\frac{Y(s)}{U(s)} = \frac{Y(s)}{U(s)}$$

$$Y(s) = CX(s)$$

$$\frac{Y(s)}{U(s)} = C[SI - A]^{-1}B$$

$$T.F. = C[SI - A]^{-1}B$$

$$\text{adj} = \begin{bmatrix} s-3 & 1 \\ 4 & s-2 \end{bmatrix}$$

$$[SI - A]^{-1}$$

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} s-2 & -1 \\ -4 & s-3 \end{bmatrix}$$

$$\det = (s-2)(s-3) - 4$$

$$= s^2 - 5s + 6 - 4$$

$$= s^2 - 5s + 2$$

$$T.F. = C[SI - A]^{-1}B$$

$$T.F. = \frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s-3 & 1 \\ 4 & s-2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s^2 - 5s + 2} = \frac{\begin{bmatrix} s-3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s^2 - 5s + 2} = \boxed{\frac{1}{s^2 - 5s + 2}}$$

$$s = \frac{5 \pm \sqrt{25 - 4(2)}}{2}$$

$$s = \frac{5}{2} \pm \frac{\sqrt{17}}{2}$$

$$s = 4.56 \text{ or } 0.44$$

the
root

-ve
root

Canonical

$$A = \begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Q2

$$e^{At} = P e^{Dt} P^{-1}$$

$$e^{At} = P^{-1} [(sI - A)^{-1}]$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

then also use P.F. $\frac{1}{s+a} = e^{-at}$

$$\text{adj.} = \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \quad \det = s^2 + 3s + 2 = (s+2)(s+1)$$

must use adjoint.
det.

$$e^{At} = P^{-1} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \right]^{-1} = P^{-1} \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}^{-1} = P^{-1} \begin{bmatrix} \frac{s+3}{(s+2)(s+1)} & \frac{+1}{(s+2)(s+1)} \\ \frac{-2}{(s+2)(s+1)} & \frac{s}{(s+2)(s+1)} \end{bmatrix}$$

4

$$\frac{s}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$= \frac{2}{s+2} + \frac{-1}{s+1}$$

$$= 2e^{-2t} - e^{-t}$$

$$A = \left. \frac{s}{s+1} \right|_{s=-2} = \frac{-2}{-1} = 2$$

$$B = \left. \frac{s}{s+2} \right|_{s=-1} = \frac{-1}{1} = -1$$

$$\frac{+1}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$= -e^{-2t} + e^{-t}$$

$$A = \left. \frac{+1}{s+1} \right|_{s=-2} = \frac{+1}{-1} = -1$$

$$B = \left. \frac{+1}{s+2} \right|_{s=-1} = \frac{+1}{1} = +1$$

$$(sI - A)^{-1}$$

$$\frac{-2}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$= +2e^{-2t} - 2e^{-t}$$

$$A = \left. \frac{-2}{s+1} \right|_{s=-2} = \frac{-2}{-1} = +2$$

$$B = \left. \frac{-2}{s+2} \right|_{s=-1} = \frac{-2}{1} = -2$$

$$\frac{s+3}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$= -e^{-2t} + 2e^{-t}$$

$$A = \left. \frac{s+3}{s+1} \right|_{s=-2} = \frac{-1}{-1} = +1$$

$$B = \left. \frac{s+3}{s+2} \right|_{s=-1} = \frac{2}{1} = 2$$

$$\therefore e^{At} = \begin{bmatrix} -e^{-2t} + 2e^{-t} & -e^{-2t} + e^{-t} \\ -2e^{-2t} - 2e^{-t} & 2e^{-2t} - e^{-t} \end{bmatrix} \checkmark$$

Q3) given $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

a) $G = \begin{bmatrix} B & AB & A^2B & \dots \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 4 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\det G = -1 \checkmark \neq 0$$

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \end{bmatrix}$$

$$CA = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

$$O = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \det(O) = 1 \checkmark \neq 0$$

b)

P.O. = 5%
 $T_s = 2 \text{ sec}$

$\xi = 0.6401$
 $\tau_n = 2.896$

given $u = -KX + lr$

$r = \text{unit step}$

$$\dot{X} = AX + BU$$

$$Y = CX$$

$$s^2 + 4s + 8.4 = 0$$

So find K ?

$$A - BK = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3-k_1 & 4-k_2 \end{bmatrix}$$

$$\det [sI - (A - BK)]$$

$$\det \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3-k_1 & 4-k_2 \end{bmatrix} \right) = \det \begin{bmatrix} s-2 & -1 \\ k_1-3 & s+4-k_2 \end{bmatrix} = (s-2)(s+4-k_2) + (k_1-3)$$

$$= s^2 + k_2s - 4s - 2s - 2k_2 + 8 + k_1 - 3$$

$$= s^2 + s(k_2 - 6) + k_1 - 2k_2 + 5$$

$$k_2 - 6 = 4$$

$$k_2 = 10$$

$$\therefore K = \begin{bmatrix} 23.4 & 10 \end{bmatrix}$$

$$k_1 - 2k_2 + 5 = 8.4$$

$$k_1 = 8.4 - 5 + 2(10)$$

$$k_1 = 23.4$$

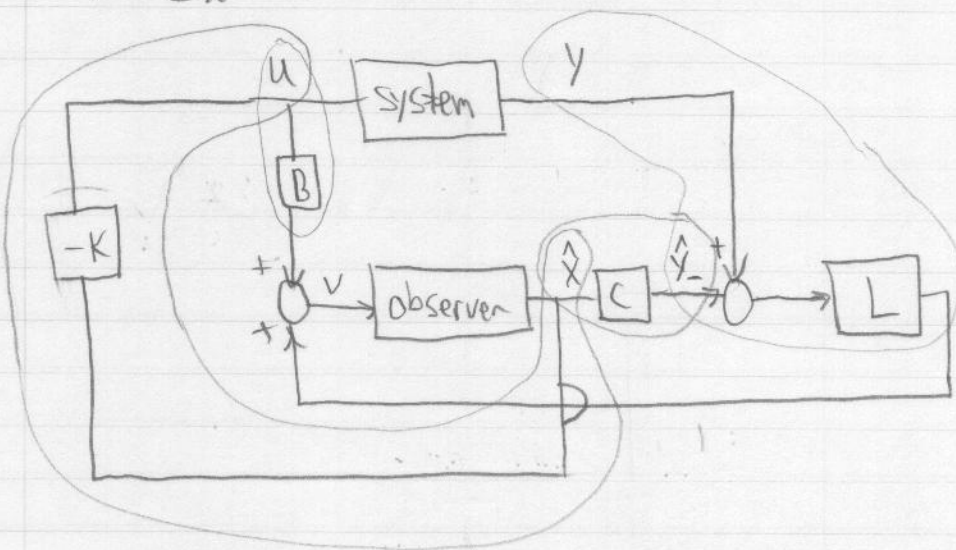
2004 #7

c) Observer Based State Feedback

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$u = -K\hat{x}$$

$$\hat{y} = C\hat{x}$$



State feedback controller plus integrator

$$y = Cx$$

